PROMPT AND DELAYED RADIO BANGS AT KILOHERTZ BY SN 1987A: A TEST FOR GRAVITON-PHOTON CONVERSION

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Abstract

A sequence of prompt and delayed radio signals at tens of kilohertz should reach the Earth (or Jupiter) due to graviton–photon conversion in interstellar as well as local planetary magnetic fields. These radio fluxes may be a detectable probe of a huge gravitational burst expected from Supernovae explosions. The earliest prompt radio signal, coinciding with the neutrino burst, is due to conversion in the terrestrial (or Jovian) magnetic field and is below the micro-Jansky (or milli-Jansky) level for a galactic Supernova like SN1987A. A later radio signal, a “tail”, due to the same graviton - radio wave conversion in random interstellar fields will maintain a relic radio “noise” for hundreds or thousands of years and might even be still detectable by a very sensitive network of satellite antennas at the kilohertz band. Exact solutions are presented here for the graviton-photon conversion in a refractive medium, as well as their consequences for high energy supernovae and the 2.726 K background radiation.

1 Introduction: the graviton–photon conversion

When a massless graviton interacts with an orthogonal component of a stationary electromagnetic field, it may decay into a pair of massless photons: a real photon with almost the same graviton wavevector and frequency, and a virtual one due to the external stationary field. Since Gravitational Waves (GW) and Electromagnetic Waves (EW) propagate in perfect vacuum at the speed of light c, the processes reinforce themselves, leading to resonant phenomena discovered

first by Gertsenshtein in 1961 [1]. There is an analogous and better known process in general relativity, with a Feymann diagram describing photon-photon scattering by a virtual graviton: the deflection of light by gravitational field which made so popular, since 1919, the Einstein theory. However, contrary to light deflection, graviton-photon conversion has been never observed and may have deeper consequences in future GW astronomy. In classical field theory one may understand graviton-photon conversion as a result of a background metric perturbation in a stationary magnetic or electric field: the space-time vibration of GWs squeezes the magnetic or electric field lines, which themselves become more and more, all along the GW propagation, sources of real EWs [2,3]. The GW-EW conversion is a reversible process. Indeed, a real photon in the presence of an external magnetic (or electric) field may be annihilated by a virtual photon, creating a real graviton. This reverse process could be seen from classical field theory as a constructive interference between free EW and the stationary electromagneticel field: this interference is the source of an "energy beating component" along the EW propagation path, which reinforces itself and becomes a "resonant" source of a GW flying in the same direction. At a very long distance the full process is oscillatory, but for most realistic astrophysical and even cosmological cases the conversion occurs only at a very limited size; then it may be important to consider only partial conversion from one (GW) to the other (EW) form of energy, with the efficiency depending quadratically on the distance. This reversible dimensionless energy conversion efficiency α is equal in both directions (GW $\leftrightarrow$ EW), and in perfect vacuum, in the first approximation, was found [1,6] to be

$$\alpha_a = \frac{P_{\rm GW}}{P_{\rm EW}} = \frac{GB^2L^2}{c^4} = 8.26 \cdot 10^{-25} \left(\frac{B}{10^6 G} \; \frac{L}{30 {\rm km}} \right)^2 . \tag{1}$$

Here $P_{\rm GW}$ and $P_{\rm EW}$ stand for the GW and EW powers.

In what follows we shall consider for real cases only stationary magnetic fields because, as Nature and consequentely the Maxwell equations teach us (and Parker's bounds on monopoles imply), we may expect only large-size coherent magnetic fields due to the total (or negligible) absence of free magnetic monopoles.

Therefore $B \equiv |B_\perp|$ is a stationary magnetic field orthogonal to the free GW (or EW) and L is the path length crossed by GW (or EW) under the stationary field B; G and c are the Newtonian constant and the velocity of light, respectively.

When the GW-EW conversion takes place in an oscillatory manner (in perfect vacuum), the conversion efficiency becomes α_0:

$$\alpha_0 = \frac{P_{\rm GW}}{P_{\rm EW}} = \tan^2 \left[\frac{G^{1/2}BL}{c^2} \right] \simeq \left(\frac{G^{1/2}BL}{c^2} \right)^2 = \alpha_a . \tag{2}$$

The last approximate relation holds only when $\alpha_0 \ll 1$, as was assumed in equa-

tion (1). The smallness of the above conversion efficiency makes the phenomena very difficult to observe in laboratory. Moreover, the presence of any realistic refractive index (in a terrestrial laboratory or in an astrophysical or even cosmological framework) often reduces the efficency, or, worse, it may dilute its arrival in time, leading to a serious problem concerning the signal-to-noise ratio. Therefore it is more convenient to disregard the very difficult double conversion (photon-graviton-photon) in laboratory and to consider only the GW-EW conversion in an astrophysical framework.

The presence of a refractive index will lead to a sequence of EW $\leftrightarrow$ GWs conversions, an incoherent "multiconversion" which enhances the oscillatory conversion.

One could treat as a GW source one of the few known periodic sources, such as the binary system PSR 1913+16. However, their frequency is very low, so that the refractive index of free electrons in the interstellar space makes the GW-EW conversion hopelessly weak.

Therefore the best sources we are able to suggest for the GW-EW conversion are the galactic Supernovae which might emit at higher total power and at much larger frequency, so that the refractive index of electrons might be less severe in screening or delaying the GW $\leftrightarrow$ EW conversion.

Moreover, the local terrestrial or Jovian magnetic fields might also convert SN GWs as soon as neutrino bursts arrive, and therefore it would be easier to observe them in coincidence with time-direction constraints. Extended interstellar magnetic fields may give life to much longer delayed radio tails. This occurs in the presence of free intergalactic charges, because the effective "mass of radio photons" makes the photons travel at a speed smaller than c. Therefore the radio signal will come from long distances later and later, leading to a diluted radio signal. Their continuous conversion signals will produce a long (thousands of years) delayed radio noise which might be more difficult to observe.

The GW conversion cannot take place too near a supernova where the magnetic fields are the greatest, because of the huge ionization (and hence a huge refractive index) due to the explosion. Unfortunately, the stellar and sky radio noise at tens of kilohertz may be dominated by local sources of noise which can hide the simplest GW $\leftrightarrow$ EW conversion in the perfect case.

Finally, the EW $\leftrightarrow$ GW conversion may also affect (but at much lower level than the present sensitivity) the cosmic background rediation, at a temperature perturbation level $\Delta T/T \simeq 10^{-7}$.

2 Gravitational and electromagnetic field equations in stationary fields

The photon-graviton conversion phenomenon has been first discovered by Gertsenhtein [1] in 1961. This process is a secondary effect of gravitational syn-

chrotron radiation [2,6] and has been also analyzed by different authors [3,6]. Let us briefly reconsider the equations of the GW $\leftrightarrow$ EW oscillations. Following Landau and Lifshits's notation [7], we consider a nearly flat space-time η_{ik} with a small metric perturbation h_{ik}; we also introduce the traceless tensor ψ^i_k:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}; \qquad h_{\mu\nu} = \psi_{\mu\nu} - \frac{1}{2}\psi^\sigma_\sigma g^0_{\mu\nu}, \tag{3}$$

so that the Einstein field equations $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ in the linear approximation become

$$\Box\psi^\mu_\nu = \Box h^\mu_\nu = -\frac{16\pi G}{c^4}\tau^\mu_\nu \tag{4}$$

where τ^μ_ν is the energy momentum tensor (the first equality in (4) holds because $\psi^\mu_\mu = 0$). In an external stationary electromagnetic field $F^{\sigma\tau(o)}$ and in the presence of a free EW, $\tilde{F}^{\sigma\tau}$, the total electromagnetic (EM) field is

$$F^{\mu\nu} \equiv F^{\mu\nu(0)} + \tilde{F}^{\mu\nu} \tag{5}$$

The corresponding energy-momentum tensor is

$$\tau^\mu_\nu = \frac{1}{4\pi}\left[F^{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta^\mu_\nu\left(F^{\sigma\tau}F_{\sigma\tau}\right)\right] \tag{6}$$

This expression may be considered to consist of three independent components: the first one proportional to constant terms, $\left(F^{\mu\sigma(0)}F_{\mu\nu(0)}\right)$, the second one proportional to the free field terms, $\left(\tilde{F}^{\mu\nu}\tilde{F}_{\mu\tau}\right)$ describing the massless EW (or just photons) and the third one proportional to the free-stationary field interference: $2\tilde{F}^{\mu\nu}F^{(o)}_{\mu\sigma}$.

The first term is not a GW source since it is constant, the second one cannot be a source because of the energy-momentum conservation law of free massless particles, while the third one is resonant and can be an effective source of GWs. Indeed, the presence of an external (virtual photon) field allows the necessary momentum to be transferred outside the free massless system. Then we can write the effective Einstein equations as follows:

$$\begin{aligned}\Box h^\mu_\nu &= -\frac{16\pi G}{c^4}\tau^\mu_\nu = \\ &= -\frac{8G}{c^4}\left[F^{(0)\mu\sigma}\tilde{F}_{\nu\sigma} - \frac{1}{4}\delta^\mu_\nu\left(F^{(0)\sigma\tau}\tilde{F}_{\sigma\tau}\right)\right]\end{aligned} \tag{7}$$

Due to the absence of magnetic monopoles we shall consider mainly large-size magnetic fields. For simplicity let us consider a uniform stationary magnetic field B_{0z} along the z axis and an orthogonal EW, polarized with an induction vector $\tilde{B}_z$, parallel to B_{0z} and propagating along the x axis. In this case the tensor components of the wave equations (8) reduce to

$$\begin{gathered}\Box h^1_1 = \Box h^2_2 = \Box h^1_0 = (4G/c^4)B_{0z}\tilde{B}_z, \\ \Box h^3_3 = \Box h^0_0 = \Box h^0_1 = -(4G/c^4)B_{0z}\tilde{B}_z;\end{gathered} \tag{8}$$

for the other metric components one finds

$$h_2^2 = -h_3^3, \quad h_2^3 = h_3^2 = h_2^3 = 0. \tag{9}$$

Moreover, for simplicity, we may assume the EW to be a plane linearly polarized wave

$$\tilde{B}_z \equiv \tilde{B}_{z0}\, e^{i(kx-\omega t)} \qquad (|\tilde{E}_y| = |\tilde{B}_z|). \tag{10}$$

Neglecting the feedback reaction (of GW on EW), one can solve Eq. (8) [1] assuming a slowly varying function $h_2^2 = b(x)$, $\left(d^2b/dx^2 = 0\right)$. This procedure leads to a conversion factor α_a of Eq. (1); in general one must also include the reverse process, i.e., EW production by GW. Let us evaluate this general behaviour. The Maxwell equations in a vacuum curved space-time

$$F_{\mu\nu;\sigma} + F_{\sigma\mu;\nu} + F_{\nu\sigma;\mu} = 0 \tag{11}$$

may be reduced in the present case to a few relevant components by substituting the covariant derivative definition ($F_{\mu\nu;\sigma} = F_{\mu\nu,\sigma} - \Gamma^\tau_{\mu\sigma} F_{\tau\nu} - \Gamma^\tau_{\nu\sigma} F_{\nu\tau}$) [3,4]:

$$F^{21}_{,0} + F^{20}_{,1} = -B_{oz} h_{22,1}, \quad F^{21}_{,1} + F^{20}_{,0} = 0. \tag{12}$$

Taking a space derivative in the first equation and a time derivative in the second one, we reduce Eqs. (8) and (12) to a set of wave equations:

$$\begin{aligned} \Box \tilde{B}_z = -B_{0z} h_{22,11} = B_{0z} k^2 h_{22}, \\ \Box h_2^2 = (4G/c^4) B_{0z} \tilde{B}_z. \end{aligned} \tag{13}$$

This set may be written in a more symmetric form by defining an energy density amplitude for both GW and EW [8]. These energy densities are

$$\begin{aligned} \rho_{\mathrm{EW}} &= \frac{\tilde{B}_z^2}{8\pi} + \frac{\tilde{E}_y^2}{8\pi} = \frac{\tilde{B}_z^2}{4\pi} \\ \rho_{\mathrm{GW}} &= \frac{c^2}{16\pi G}[\dot{h}_{23}^2 + \frac{1}{4}(\dot{h}_{22} - \dot{h}_{33})^2] \end{aligned} \tag{14}$$

where a dot stands for a time derivative. From the set (14) we may define each energy amplitude, $\rho_{\mathrm{EW}} \equiv a^2$ and $\rho_{\mathrm{GW}} \equiv b^2$:

$$a \equiv \tilde{B}_z/\sqrt{4\pi}, \quad b \equiv c\omega h_2^2/\sqrt{16\pi G} \tag{15}$$

where $h_2^2 = -h_3^3 = h_{22}$. Substituting these amplitudes to the set of wave equations, one finds a simpler one:

$$\Box a = (k^2c^2/\omega^2) \cdot pb \simeq pb, \quad \Box b = pa \tag{16}$$

where $p \equiv (2\omega/c^3) B_{0z} (G)^{1/2}$. The last approximation in the first equation of the above set holds when $kc \simeq \omega$. Therefore, strictly speaking, the photon-graviton oscillation is not exactly symmetric (contrary to what was assumed

in Ref. [8]). Moreover, the system should be generalized to take care also of the EW dispersion law due to the dielectric behaviour of neutral matter or due to plasma conductivity, as well as quantum electrodynamic corrections to the Lagrangian (the Feymann box diagrams due to radiative corrections which allow photon-photon scattering).

3 Generalized Zel'dovich dispersion law for the graviton-photon oscillation in a refractive medium

Eqs. (16), describing the photon-graviton oscillations, can contain different refractive terms; the main ones, due to classical and quantum electrodynamics, are given by the following wave equation:

$$\Box|\tilde{B}_z \equiv \left[k^2 - \left(\mu\epsilon\frac{\omega^2}{c^2} + 4\pi i\frac{\mu\omega\sigma}{c^2}\right) - \left(\frac{e^2}{\hbar c}\right)^2 \frac{\hbar^3}{m^2c^5}\left(\frac{\omega}{c}\right)^2 B_{0z}^2\right]\tilde{B}_z = B_{0z}k^2h_2^2 \tag{17}$$

where the symbol $\Box|$ stands for a generalized D'Alambert operator.

The corresponding EW $\leftrightarrow$ GW conversion will be no longer symmetric and the equations must be rewritten as follows:

$$\begin{aligned} \Box|a &\equiv \Box a - (r_a + r_e + r_B)a = p(k^2c^2/\omega^2)b \simeq pb, \\ \Box|b &\equiv \Box b - qb = pa \end{aligned} \tag{18}$$

where r_a, r_e and r_b are the refractive terms related to those in Eq. (17) (which are proportional to $n^2 - 1$ where n is the refractive index), connecteed with atomic polarization (r_a), the plasma conductivity (r_e) and the nonlinear behaviour of Q.E.D. (r_B).

The similar refractive term q is due to the presence of another form of energy density perturbed by GWs themselves: $q \simeq (G/c^4)\rho_{em}$. The refractive terms r and q and the conversion factor p are considered below in a realistic framework of astrophysical and cosmological interest: in particular, we analyze the EW $\leftrightarrow$ GW conversion in the infrared optical band in laboratory, as well as the reverse phenomenon [6]. Therefore we shall further discuss:

(1) GW emitted by supernovae to the interstellar space and their conversion GW $\rightarrow$ EW within the kilohertz band;

(2) EW of the cosmological background radiation spectrum at millimeter wavelengths and their deformation due to the conversion into GW by cosmological and galactic magnetic fields.

We report here the characteristic values of the refractive terms for astro-

physical and cosmological problems:

$$\begin{array}{lll}
\mathit{Definition} & \mathit{Astrophysical} & \mathit{Cosmological} \\
r_a \equiv \frac{\omega^2}{c^2}\left(\frac{\omega_{pa}^2}{\omega_0^2-\omega^2}\right) & \simeq 10^{-40}\left(\frac{n_a}{10^{-1}cm^{-3}}\right) & = 10^{-29}\left(\frac{n_a}{10^{-6}cm^3}\right) \\
 & \left(\frac{\omega}{3\cdot 10^3\ \mathrm{Hz}}\right)^2\left(\frac{\omega_0}{6\cdot 10^{14}\mathrm{Hz}}\right)^{-2} cm^{-2} & \left(\frac{\omega}{3\cdot 10^{11}\mathrm{Hz}}\right)^2\left(\frac{\omega_0}{6\cdot 10^{14}\mathrm{Hz}}\right)^{-2} cm^{-2} \\
r_e \equiv -\frac{\omega_p^2}{c^2} = -\frac{4\pi n_e e^2}{m_e c^2} & = -8\cdot 10^{-14}\left(\frac{n_e}{cm^{-3}}\right)\ cm^{-2} & -8\cdot 10^{-24}\left(\frac{n_e}{10^{-10}cm^{-3}}\right) cm^{-2} \\
r_{e\pm} \equiv \frac{\omega_p^2}{c^2}\frac{\omega}{\omega\pm\omega_B} & & \\
r_{e+} \equiv \frac{\omega^2}{c^2}\frac{\omega}{\omega+\omega_B} & = 10^{-18}\left(\frac{B}{2G}\right)^{-1}\left(\frac{n_e}{cm^{-3}}\right)\left(\frac{\omega}{10^3\ \mathrm{Hz}}\right) & \\
r_B \equiv \left(\frac{e}{\hbar c}\right)^2\frac{\hbar^3}{m^2c^5}\left(\frac{\omega^2}{c^2}\right)B_{z0}^2 & = 3\cdot 10^{-44}\left(\frac{B_{0z}}{G}\right)^2\left(\frac{\omega}{3\cdot 10^3\ \mathrm{Hz}}\right)^2 cm^{-2} & = 3\cdot 10^{-46}\left(\frac{B_{0z}}{10^{-9}}\right)^2\left(\frac{\omega}{3\cdot 10^{11}\mathrm{Hz}}\right)^2 cm^{-2} \\
q = \frac{G}{c^4}\left(\frac{B^2}{4\pi}+\rho_r\right) & = 7\cdot 10^{-51}\left(\frac{B}{\mathrm{Gauss}}\right)^2 cm^{-2} & \\
p \equiv 2\frac{\omega}{c}\frac{\sqrt{G}}{c^2}B_{0z} & 5\cdot 10^{-32}\left(\frac{\omega}{3\cdot 10^3\ \mathrm{Hz}}\right)\left(\frac{B}{\mathrm{Gauss}}\right) cm^{-2} & = 5\cdot 10^{-33}\left(\frac{\omega}{3\cdot 10^{11}\mathrm{Hz}}\right)\left(\frac{B_0}{10^{-9}\mathrm{Gauss}}\right) cm^{-2}
\end{array} \tag{19}$$

where n_a and n_e are, respectively, the neutral gas and electron number density, $\omega_{\rm pa} = 4\pi n_a e^2/m_a$ is written for hydrogen, ω_0 is the ionization frequency for hydrogen, ρ_r is the EM radiation energy density in the propagation medium. All these quantities are written in the units related to the problem under consideration.

A generalization of the refractive term r_e ($r_{e\pm}$) is needed due to the dipolar nature of the propagation of EW in magnetic fields; the two extreme cases of circular polarized modes (the birefringent modes [9]) are shown in (19) for the typical terrestrian magnetic field; $\omega_B = eB/(mc) = 1.6\cdot 10^7(B/\mathrm{Gauss})\mathrm{Hz}$ and $\omega \sim 10^3 h \ll \omega_B$. These refractive values are a special solution (in the equatorial plane) of a generalized Appleton-Hartree dispersion equation [10] whose solutions $r_e(\theta)$ are complicated functions of the polar angle.

From a rapid inspection of the conversion term p and the refractive term r one notices that in general $|r| \gg |p|$, and the refractive medium cannot be neglected in principle. From the differential equations (18) and for the quantities defined in (19) we can evaluate exact and approximated solutions for the GW $\leftrightarrow$ EW conversion. We may in general neglect the GW refractive term q because

$$q \ll p < |r| \tag{20}$$

Therefore the set (18) reduces to

$$\begin{aligned}
\Box a &= ra + k^2\left(\frac{\omega}{c}\right)^{-2} pb \simeq ra + pb, \\
\Box b &= pa
\end{aligned} \tag{21}$$

where $r \equiv r_a + r_e + r_B$ and $\Box \equiv -\partial^2/\partial x^2 + c^{-2}\partial^2/\partial t^2$.

The approximation of Eq. (21) holds because in general $k \simeq \omega/c$, i.e., $(\omega/c)^2 \gg r,\ p$. Assuming for the EW and GW energy density amplitudes a plane wave solution of the form

$$a = a_0 e^{i(kx-\omega t)}, \quad b = b_0 e^{i(kx-\omega t)}, \tag{22}$$

one finds from Eq. (21) the exact generalized Zel'dovich dispersion law

$$
\begin{aligned}
k_{\pm}^2 &= \frac{\omega^2}{c^2} + \frac{r}{2} + \frac{p^2}{2(\omega/c)^2} \\
&\pm \left[\left(\frac{\omega^2}{c^2} + \frac{r}{2} + \frac{p^2}{2\omega^2/c^2} \right)^2 - \frac{\omega^2}{c^2} \left(\frac{\omega^2}{c^2} + r \right) \right]^{1/2} .
\end{aligned} \tag{23}
$$

This equation has 4 roots: we will restrict ourselves to forward travelling waves, i.e., consider only positive k:

$$
\begin{aligned}
k_{\pm} &= |(k_{\pm}^2)^{1/2}| = \frac{\omega}{c} \left\{ 1 + \frac{r}{2\omega^2/c^2} + \frac{p^2}{2\omega^4/c^4} \right. \\
&\pm \left. \left[\left(1 + \frac{r}{2\omega^2/c^2} + \frac{p^2}{2\omega^4/c^4} \right)^2 - \left(1 + \frac{r}{\omega^2/c^2} \right) \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} .
\end{aligned} \tag{24}
$$

Note that negative wave vectors, corresponding to reflected GW, are also a very interesting phenomenon which may offer a decisive indication of the EW $\leftrightarrow$ GW oscillation. However, the back gravitational reflection, either in vacuum [4], or in the presence of a refractive medium, is strongly suppressed as compared with the advancing wave. Finally, the backward conversion process is no longer quadratically dependent on the distance L but depends in this way just on the wavelength λ and is consequentely drastically suppressed as compared with the usual forward conversion. Moreover, the presence of a reflective index will lead to a "massive" photon and to a consequent delay of its arrival with respect to the corresponding gravitons. A similar delay from the SN has been considered as a tool for measuring the neutrino mass [11]. Such a delay would dilute and spoil the EW radio bang signals by supernovae, as discussed in the Conclusion.

4 Exact solutions for the generalized Zel'dovich dispersion law

In general, for advanced waves the energy density amplitudes may be written as follows:

$$
\begin{aligned}
a &= a_+ e^{i(k_+ x - \omega t)} + a_- e^{i(k_- x - \omega t)}, \\
b &= b_+ e^{i(k_+ x - \omega t)} + b_- e^{i(k_- x - \omega t)}.
\end{aligned} \tag{25}
$$

For $r = 0$, $k_{\pm} = \frac{\omega}{c} \left[1 \pm \frac{p}{(2\omega/c)^2)} \right]$. Each eigenvalue of the wave vector $k_{\pm}$ in Eq. (24) corresponds to an eigenvector $a_{\pm}$, $b_{\pm}$. To find them, it is sufficient to consider the approximate Eq. (21)

$$
\Box a \simeq ra + pb \tag{26}
$$

valid for $(\omega/c)^2 \gg r,\ p$, i.e., in most real cases. Then

$$a_\pm = \frac{p}{\lambda_\pm - r} b_\pm = \frac{\lambda_\pm}{p} b_\pm \tag{27}$$

where $\lambda_\pm = k_\pm^2 - \omega^2/c^2$ are solutions to the eigenvalue equations (21):

$$\begin{aligned} \lambda^2 - \lambda\, r - p^2 = 0 \\ \lambda_\pm = \frac{1}{2} r \left(1 \pm \sqrt{1 + 4p^2/r^2}\right). \end{aligned} \tag{28}$$

Let us reconsider two extreme cases: $(\omega/c)^2 \gg p \gg r$ (almost perfect vacuum) and $(\omega/c)^2 \gg r \gg p$ (real refractive medium). The wave vector difference Δk is

$$\begin{aligned} \Delta k &\equiv k_+ - k_- \simeq \\ &\simeq \frac{\omega}{c} \left(\frac{r^2}{4(\omega/c)^4} + \frac{p^2}{(\omega/c)^4} + \frac{p^4}{4(\omega/c)^8} + \frac{rp^2}{2(\omega/c)^6} \right)^{1/2} \end{aligned} \tag{29}$$

and for $|p| \gg |r|$ one finds the limiting values:

$$\begin{aligned} k_\pm^2 &= \frac{\omega^2}{c^2} \left[1 + \frac{p^2}{2(\omega/c)^4} + \frac{r}{2(\omega/c)^2} \right. \\ &\quad \left. \pm \frac{p}{(\omega/c)^2} \left(1 + \frac{r^2}{4p^2} + \frac{p^2}{4(\omega/c)^4} + \frac{r}{2(\omega/c)^2} \right)^{1/2} \right], \\ \Delta k &\simeq \frac{p}{(\omega/c)} \left[1 + \frac{r^2}{8p^2} + \frac{p^2}{8(\omega/c)^4} + \frac{r}{4(\omega/c)^2} \right], \\ \lambda_\pm &= \pm p \left[1 + \frac{r^2}{4p^2} \right]^{1/2} + \frac{r}{2} = \pm p \left(1 + \frac{r^2}{8p^2} \right) + \frac{r}{2}, \\ \frac{\lambda_-}{\lambda_+} &\simeq \frac{-p + r/2}{p + r/2} \simeq -1 - \frac{r}{p}. \end{aligned} \tag{30}$$

When $|r| \gg p$:,

$$\begin{aligned} k_\pm^2 &= \frac{\omega}{c} \left[1 + \frac{r}{2(\omega/c)^2} + \frac{p^2}{2(\omega/c)^4} \right. \\ &\quad \left. + \frac{p^2}{2r(\omega/c)^2} + \frac{p^2}{8(\omega/c)^2} \right], \\ \Delta k &\simeq \frac{|r|}{2(\omega/c)} \left[1 + \frac{2p^2}{r^2} + \frac{p^4}{2(\omega/c)^4 r^2} + \frac{p^2}{r(\omega/c)^2} \right], \end{aligned} \tag{31}$$

then for $r > 0$

$$\lambda_\pm = \frac{r}{2} \left[1 \pm \left(1 + \frac{4p^2}{r^2} \right)^{1/2} \right] \simeq \begin{cases} r\left(1 + \frac{p^2}{r^2}\right), \\ -\frac{p^2}{r}, \end{cases} \tag{32}$$

while for $r < 0$

$$\lambda_\pm = \frac{|r|}{2}\left[-1 \pm \left(1 + \frac{4p^2}{r^2}\right)^{1/2}\right] \simeq \begin{cases} \frac{p^2}{|r|}, \\ -|r| - \frac{p^2}{|r|}; \end{cases} \tag{33}$$

$$\begin{aligned}
\frac{\lambda_-}{\lambda_+} &\simeq \frac{-p^2/r^2}{1 + rp^2/r^2} = \frac{-p^2}{p + r^2} \quad for \;\; r > 0, \\
\frac{\lambda_-}{\lambda_+} &\simeq \frac{-|r|(1 + p^2/r^2)}{p^2/|r|} = \\
&= \frac{-r^2}{p^2}\left(1 + \frac{p^2}{r^2}\right) = -\frac{r^2}{p^2} - 1, \;\; for \;\; r < 0.
\end{aligned} \tag{34}$$

Given these expressions for Δk, $\lambda_\pm$, and the eigenvector relations of Eq. (27) in the needed approximations, we can easely analyze the GW $\leftrightarrow$ EW conversion for any realistic framework in physics, astrophysics and cosmology.

5 Conversion efficiency in a single oscillatory period

Let us first consider as initial conditions ($t = 0$, $x = 0$) a vanishing GW ($b_{in} = 0$) and a strong EW beam propagating in an orthogonal stationary field B_{0z} ($a_{in} \neq 0$) [6]; for instance, the EW may be a laser beam along a magnetized tunnel where $B_{0z} \parallel \tilde{B}_z$, i.e., where the stationary field is parallel to the polarized EW $\tilde{B}$ field. The corresponding GW energy density amplitude b is zero at $t = 0$, $x = 0$, while the EW amplitude a has a given value a_{in}. From Eq. (25) one obtains:

$$b_+ = -b_-, \quad a_\pm = \frac{\lambda_\pm}{p} b_\pm, \quad \frac{a_+}{a_-} = -\frac{\lambda_+}{\lambda_-}. \tag{35}$$

From Eq. (25) we thus obtain

$$\begin{aligned}
a &= a_+\left[e^{i(k_+x - \omega t)} - \frac{\lambda_-}{\lambda_+} e^{i(k_-x - \omega t)}\right], \\
b &= b_+\left[e^{i(k_+x - \omega t)} - e^{i(k_-x - \omega t)}\right] = \\
&= 2a_+ \frac{p}{\lambda_+} i \left[\sin\frac{\Delta\kappa x}{2}\right] e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]}
\end{aligned} \tag{36}$$

where $k_\pm$, $\lambda_\pm$, Δk are defined in (24), (28), (29); in the case $p \gg |r|$ we obtain from Eq. (30) the explicit solutions

$$\begin{aligned}
a &= 2a_+ \cos\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]} + \frac{r}{p}\, a_+ e^{i(k_-x - \omega t)}, \\
b &= \frac{2a_+ i}{1 + r/(2p)} \sin\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]}.
\end{aligned} \tag{37}$$

For $r = 0$ the solution of Eq. (37) reduces to the perfect vacuum case:

$$a = 2a_+ \cos\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]}, \quad b = 2a_+ i \, \sin\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]}. \quad (38)$$

In the low refraction limit ($p \gg |r|$) the EW $\to$ GW conversion occurs at a rate

$$\alpha \equiv \frac{|b|^2}{|a|^2} = \frac{4}{(1 + r/2p)^2} \frac{\sin^2(\frac{1}{2}\Delta K\, x)}{\left[4\cos^2(\frac{1}{2}\Delta k\, x) + 4(r/p)\cos(\frac{1}{2}\Delta k\, x) + r^2/p^2\right]}; \quad (39)$$

in the zero refraction limit

$$\lim_{r\to 0} \alpha = \tan^2\left(\frac{\Delta k\ x}{2}\right) \simeq \frac{p^2 x^2}{4(\omega/c)^2} = \frac{G}{c^4} B_{0z}^2 x^2 \quad (40)$$

where the last approximation in Eq. (40) holds for $x \leq L_{\rm coh} = 2\pi/\Delta k = (2\pi/p)(\omega/c)$. Therefore we return to the classical conversion efficiency of Eqs. (1),(2).

From Eq. (39) one may suspect that the conversion efficiency for $r \gg p$ will be suppressed by a factor $(p/r)^2$, but this is not the case because Eq. (39) holds only under the assumption $p \gg |r|$. Indeed, for $|r| \gg p$ we should consider the two extreme possibilities $r < 0$ or $r > 0$ (i.e., $|r_e| < |r_a + r_B|$ or $|r_e| > |r_a + r_B|$). From the approximate eigenvector and eigenvalue expressions $\lambda_\pm$, Δk, λ_-/λ_+ in Eqs. (31)–(36) we easily find the EW and GW energy amplitudes, a and b, respectively: for $r > 0$ and $p \ll r$

$$\begin{aligned}
a &= a_+ \left[e^{i(k_+ x - \omega t)} + \frac{p^2}{p^2 + r^2} e^{i(k_- x - \omega t)} \right], \\
b &= 2i \frac{p a_+}{\lambda_+} \sin\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]} \\
&= 2\frac{p}{r} \frac{a_+ i}{(1 + p^2/r^2)} \sin\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]},
\end{aligned} \quad (41)$$

while for $r < 0$ and $p \ll |r|$

$$\begin{aligned}
a &= a_+ \Big[e^{i(k_+ x - \omega t)} + \frac{p^2 + r^2}{p^2} e^{i(k_- x - \omega t)} \Big] = \\
&= \frac{r^2}{p^2} a_+ \Big[\frac{p^2}{r^2} e^{i(k_+ x - \omega t)} + \frac{p^2 + r^2}{p^2} e^{i(k_- x - \omega t)} \Big]; \\
b &= 2i \frac{p a_+}{\lambda_+} \sin\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]} = \\
&= 2\frac{|r|}{p} a_+ i \, \sin\frac{\Delta k x}{2} e^{i[\frac{1}{2}(k_+ + k_-)x - \omega t]}.
\end{aligned} \quad (42)$$

The energy conversion ratio for small distances $x \ll \frac{2\pi}{\Delta k} = L_{\rm coh} \simeq \frac{4\pi}{|r|}\omega/c$ is given by two α values:

$$\text{(i) for } r>0: \quad \alpha = \frac{|b|^2}{|a|^2} = \frac{4\frac{p^2}{r^2}\sin^2\frac{\Delta kx}{2}}{\left(1+\frac{p^2}{r^2}\right)^2[1+\left(\frac{p^2}{p^2+r^2}\right)^2+\frac{2p^2}{p^2+r^2}\cos\Delta kx]}; \tag{43}$$

$$\text{(ii) for } r<0: \quad \alpha = \frac{|b|^2}{|a|^2} = \frac{\frac{4p^2}{r^2}\sin^2\left(\frac{\Delta kx}{2}\right)}{[\frac{p^4}{r^4}+\left(\frac{p^2+r^2}{r^2}\right)^2+2\frac{p^2}{r^2}\left(\frac{p^2+r^2}{r^2}\right)\cos\Delta kx]} \tag{44}$$

Apparently the ratio is suppressed by a factor p^2/r^2.

However, in both solutions the conversion ratio may be approximated for $p/r \to 0$ as follows:

$$\alpha = \frac{|b|^2}{|a|^2} \simeq 4\frac{p^2}{r^2}\sin^2\frac{\Delta kx}{2} \simeq 4\frac{p^2}{r^2}\,\frac{r^2}{16(\omega/c)^2} = \frac{G}{c^4}B_{0z}^2x^2, \tag{45}$$

i.e., even when $|r|>p$, but as long as $x \ll 2\pi/\Delta k$, the conversion factor is as large as in the perfect vacuum case ($|p| \gg |r|$). This surprising result may be understood as follows: in vacuum the conversion ratio is not suppressed but the corresponding coherence length $2\pi/\Delta k$ (for full conversion) is very large:

$$L_{\rm coh}(p \gg r) = \frac{2\pi}{\Delta k} = \frac{2\pi}{p}\,\frac{\omega}{c} = \frac{\pi c^2}{\sqrt{G}B_{0z}} = 1.3\cdot 10^{19}\Big(\frac{B_{0z}}{10^6 G}\Big)^{-1}\ cm \tag{46}$$

In the presence of a refractive term r the conversion efficiency α is suppressed by a large factor $(p/r)^2$ but the coherence length (for full conversion) is now much shorter as compared with the previous one just by the same correcting factor:

$$L_{\rm coh}(p \ll |r_e|) = \frac{2\pi}{\Delta k} = \frac{4\pi}{r}\,\frac{\omega}{c} = 2\frac{p}{r}L_{\rm coh}(r{=}0) = 4\cdot 10^4\left(\frac{\omega}{3\cdot 10^4{\rm Hz}}\right)^{-1}\left(\frac{B_{0z}}{G}\right)^{-2}\ {\rm km}. \tag{47}$$

This effect exactly compensates the suppression factor $(p/r)^2$ in the efficiency, leading to the same result as is valid in vacuum (Eqs. (39), (40)). In summary, the EW $\leftrightarrow$ GW conversion for $x \le L_{\rm coh}(r)$ ignores the refraction, even if $r \gg p$. However, for $x \gg L_{\rm coh}$, we should expect a new phenomenon due to wave packet separation: multiple EW $\leftrightarrow$ GW conversion.

6 Multiple EW $\leftrightarrow$ GW conversion

The amplitude solutions of the previous section are no longer valid at distances $x \gg L_{\rm coh}$. Indeed, as long as $|a| \ll |b|$ (or *vice versa*) the conversion is a monotone (rather than oscillatory) phenomenon which we call "multiple conversion". Let us describe this "multiple conversion" using an analogy: a rich man meets a

very poor friend during his walk. The two friends agree to play, while walking, a very innocent game, "the give and take game": at each finite distance ($L_{\rm coh}$), each of them gives the other a small fraction, say, $4p^2/r^2$, of his own pocket money at that moment. Let us label $|a|_0^2$ the total initial venture capital (pocket money) of the rich man and $|b|_0^2 = 0$ the corresponding initial "no money" of the poor friend. After the first distance $L_{\rm coh}$ (see Eq. (47)) the rich man gives $4p^2/r^2|a|_0^2$ to his friend, so that the rich man remains with $(1-4p^2/r^2)|a|_0^2$ (the poorer one gains at the first step $4p^2/r^2|a|_0^2$). At the next step $L_{\rm coh}$ the rich one offers a similar fraction of money $4(p^2/r^2)(1-4p^2/r^2)|a|_0^2$ but receives back only $(4p^2/r^2)^2|a|_0^2$. On the contrary, the poor one will capitalize $8(p^2/r^2)(1-2p^2/r^2)$. As long as $p/r \ll 1$, "the oscillatory give and take game" works one- way as a monotonically irreversible process (the poorer one will become richer and the rich will become poorer). Only when both players hold an equal capital, the process will go on symmetrically, in a reversible oscillatory way. To be more quantitative, let us consider the energy density evolution of the EW and GW. From Eq. (36) we can redefine the amplitude a_+ as follows:

$$a_+ = \begin{cases} \frac{p^2+r^2}{2p^2+r^2}\ a_{\rm in}, & r>0; \\ a_+ = \frac{p^2}{2p^2+r^2}\ a_{\rm in} & r<0. \end{cases} \tag{48}$$

The corresponding energy densities $\rho_{\rm EW}$ and $\rho_{\rm GW}$ become

$$\rho_{\rm EW} = |a|^2 = a_{\rm in}^2\Big(1-\frac{2p^2}{2p^2+r^2}\Big)^2\Big[1+\Big(\frac{p^2}{p^2+r^2}\Big)^2+\frac{2p^2}{p^2+r^2}\cos{(\Delta k x)}\Big] \quad r>0,$$

$$\rho_{\rm EW} = |a|^2 = a_{\rm in}^2\Big(\frac{p^2}{2p^2+r^2}\Big)^2\Big[1+\Big(\frac{p^2}{p^2+r^2}\Big)^2+\frac{2(p^2+r^2)}{p^2}\cos{(\Delta k x)}\Big], \quad r<0. \tag{49}$$

When $\Delta k\ x = \pi$, i.e., for $x = L_{\rm coh}/2 = \pi/\Delta k$, one gets

$$\begin{aligned} \rho_{\rm EW} &= \rho_{\rm in}\Big(1-\frac{2p^2}{2p^2+r^2}\Big)^2\Big(1-\frac{p^2}{p^2+r^2}\Big)^2 \simeq \Big(1-\frac{4p^2}{r^2}\Big)\rho_{\rm EW\,in}, \quad r>0; \\ \rho_{\rm EW} &= \rho_{in}\Big(\frac{2p^2}{2p^2+r^2}\Big)^2\Big(1-\frac{p^2+r^2}{p^2}\Big)^2 \simeq \Big(1-\frac{4p^2}{r^2}\Big)\rho_{\rm EW\,in}, \quad r<0. \end{aligned} \tag{50}$$

In a similar way one easily finds from Eqs. (43), (44), either for $r>0$ or $r<0$ after a distance $x = L_{\rm coh}/2$:

$$\rho_{\rm GW} = |b|^2 \simeq 4\frac{p^2}{r^2}\rho_{\rm EW\,in}. \tag{51}$$

Therefore, adding Eqs. (50) and (51), we easily verify the energy conservation law $\rho_{\rm in} = \rho_{\rm EW} + \rho_{\rm GW}$. If we want to generalize the process, we may define an average energy density $A_n = |a_n|^2$, $B_n = |b_n|^2$, i.e. the energy densities at any step n; the energy density evolution will have the form (from generalized

Eqs. (50), (51)):

$$A_n - A_{n-1} = -\frac{4p^2}{r^2}\,A_n + \frac{4p^2}{r^2}\,B_n, \qquad\qquad B_n - B_{n-1} = \frac{4p^2}{r^2}\,A_n - \frac{4p^2}{r^2}\,B_n. \quad (52)$$

Dividing by $L_{\rm coh}/2$ and assuming a continuous limit, we can write the following set of equations:

$$\begin{aligned}
\frac{dA}{dx} &\equiv \frac{A_n - A_{n-1}}{L_{\rm coh}/2} = -8\frac{p^2}{r^2 L_{\rm coh}}(A-B) = -\frac{1}{L}(A-B),\\
\frac{dB}{dx} &\equiv \frac{B_n - B_{n-1}}{L_{\rm coh}/2} = 8\frac{p^2}{r^2 L_{\rm coh}}(A-B) = \frac{1}{L}(A-B)
\end{aligned} \quad (53)$$

where the characteristic distance $L \equiv (r^2/8p^2)L_{\rm coh}$ is the relaxation length of the system (53). Assuming a solution of the form $A = A_0 e^{kx}$, $B = B_0 e^{kx}$, one finds the eigenvectors: $k_+ = 0$, $k_- = -2/L$, not to be confused with the previous ones. For the initial boundary conditions $A(t=0, x=0) = A_0$, $B(t=0, x=0) = 0$ one finally obtains:

$$\begin{aligned}
A &\equiv |a|^2 = \frac{1}{2}A_0(1+e^{-2x/L}) \simeq A_0(1-x/L),\\
B &\equiv |b|^2 = \frac{1}{2}A_0(1-e^{-2x/L}) \simeq A_0\frac{x}{L},\\
\alpha &= B/A = \tanh(x/L) \simeq x/L \simeq 8(p^2/r^2)(x/L_{\rm coh}).
\end{aligned} \quad (54)$$

The last approximations hold as long as $(r^2/p^2)L_{\rm coh} > x > L_{\rm coh}$. The average energy densities A and B are modulated by the $|a|$ and $|b|$ amplitudes in Eqs. (36)–(42) but their average value evolution in any period is given by (54). When $A \simeq A_0/2$, $B \simeq B_0/2$, the oscillatory behaviour occurs in a reversible form. The multiconversion has an analogous phenomenon in cosmology when left-handed neutrinos with Dirac and Majorana masses oscillate only in one way, leading to thermalization of the right-handed sterile neutrinos in the early Universe [12].

7 Oscillatory coherence lengths

By (46) and (47), the characteristic coherence lengths in astrophysical and cosmological problems become

$$\begin{aligned}
L_{\rm coh}(r \ll p) &= \frac{2\pi}{p}\frac{\omega}{c} = 4\cdot 10^8\Big(\frac{B}{10^6 G}\Big)^{-1}\sec\, c,\\
L_{\rm coh}(p \ll r_B) &= \frac{4\pi}{|r|}\frac{\omega}{c} = 4\cdot 10^{14}\Big(\frac{\omega}{3\cdot 10^{14}\hbar c}\Big)^{-1}\Big(\frac{B}{10^6 G}\Big)^{-2}, cm\\
L_{\rm coh}(p \ll |r_e|) &= 1.5\cdot 10^2\frac{\omega}{3\cdot 10^3 \rm Hz}\Big(\frac{n_e}{cm^{-3}}\Big)^{-1}\ \rm km
\end{aligned}$$

$$= 5 \cdot 10^4 \frac{\omega}{3 \cdot 10^{11}\text{Hz}} \frac{n_e}{cm^{-3}} \; c \cdot \text{sec},$$

$$L_{\text{coh}}(p \ll |r_{e+}|) \quad = \quad 1.2 \cdot 10^7 \frac{\omega}{3 \cdot 10^3\text{Hz}} \Big(\frac{n_e}{cm^{-3}}\Big)^{-1} \frac{B}{2 \text{ Gauss}} \text{km}. \tag{55}$$

The coherence length is defined by the minimum distance between the coherence lengths and the inhomogeneity scales of the stationary fields, magnetic or electric. For instance, we may also consider the multiconversion efficiency due to the atomic electric fields inside normal matter, where the photon and graviton wavelengths are much smaller than the atomic radius: λ_γ, $\lambda_{\tilde{g}} \ll \mathring{A}$. The random nuclear electrostatic fields become external stationary fields responsible for the EW $\leftrightarrow$ GW conversion of high energetic photons (and gravitons). The coherence length is just the atomic radius at normal densities. As a first approximation, in hydrogen, the eletric field is on average $3 \cdot 10^7$V/cm, corresponding to 10^5 Gauss. For a coherence distance $L_{\text{coh}} \simeq 1\mathring{A} \simeq 10^{-8}$ cm and for a conversion distance x of 10^7 cm one obtains:

$$\alpha \simeq 10^{-41} \Big(\frac{x}{100 \text{ km}}\Big) Z^2 \tag{56}$$

where Z is the matter atomic number. Therefore in normal matter the GW $\leftrightarrow$ EW conversion is much weaker than the coherent one for any laboratory field (in Eq. (1)) and can be neglected. For higher densities, as those in neutron stars just before neutronization, one finds much smaller coherence lengths but higher electric fields between nucleons: $L_{\text{coh}} \simeq 10^{-13}$ cm, $E \simeq 3 \cdot 10^{17}$V/cm. Therefore

$$\alpha \simeq 10^{-26} \; \frac{x}{100 \text{ km}}. \tag{57}$$

High energetic photons ($E_{\tilde{\gamma}} \geq 100$ Mev) will generate a small fraction of high energetic gravitons. We may consider incoherent multiconversion of thermal photons in thermal equilibrium in a bath of electron (or fermion, or boson) pairs (in a hot stellar core). During a supernovae explosive stage the core temperature may reach a value $kT_\gamma \geq 10 - 100$ MeV. This occurs in a region where neutrinos reach opacity (the so-called neutrino photosphere). Electron and neutrino pairs for a fraction of a second are in thermal equilibrium $T_\gamma \simeq T_{\nu_e, \tilde{\nu_e}} \simeq T_{e_\pm}$. The electron pairs (as well as photons and neutrinos) are trapped in the stellar core and are forced to run for a fraction of a second in a finite random walk for the total distance, let us say, of $x \simeq ct \simeq 3 \cdot 10^{10}$ cm. Therefore the multiconversion efficiency for thermal photons (into gravitons) in the presence of the electric field E of dense electron pair fields, at a minimal distance $r_{\min} \simeq h/kT_\gamma \simeq \lambda_\gamma t$, is

$$< \alpha > \simeq \frac{G}{c^4} < E >^2 L_{\text{coh}} x \simeq 2.4 \cdot 10^{-19} \Big(\frac{T}{10 \text{ MeV}}\Big)^3 \; \frac{x}{c \text{ sec}}. \tag{58}$$

We learn herefrom that during the SN1987A explosion a small fraction of internal energy of neutrino pairs (and photons), $E_\nu \geq 10^{53}$ erg, is converted into

a source of highly energetic gravitons $< E_{\tilde{g}} >\simeq 10 - 100$ MeV with the corresponding total energy burst

$$E_{\tilde{g}} \simeq< \alpha > E_{\nu \text{ SN}} \simeq 2.4 \cdot 10^{34} \text{ erg}, \tag{59}$$

i.e., a power in gravitons comparable to our solar power in EM waves (per second) but not so easily detectable (and much below the power of gamma burst sources). It is suggestive to consider this phenomenon as a possible explanation for energy transfer outside the collapse when neutrino opacity occurs. This result may lead to a new approch to SN explosion models. In cosmology the multiconversion may also occur at high densities and energies in the early Universe. However, the time scale of the phenomenon and therefore the corresponding flight time are also related to the temperature, namely, that in a radiation dominated Universe: $t \simeq 1$ sec $(T/\text{MeV})^{-2}$. From Eq. (58) one gets:

$$\alpha \simeq 2.4 \cdot 10^{-22} \Big(\frac{T}{MeV}\Big)^3 \ \frac{t}{\text{sec}} = 2.4 \cdot 10^{-22} \ \frac{T}{MeV}. \tag{60}$$

In a radiation dominated Universe complete conversion occurs at a critical temperature

$$T \geq 10^{22} \text{ MeV} = 10^{19} \text{ GeV} = m_{\text{pl}} c^2. \tag{61}$$

As we should expect from dimensional arguments, the photon-graviton conversion is a totally efficient process only as early as at Planck times. Therefore the photon-graviton conversion may be at least a key process in keeping gravitons in thermal equilibrium in the Universe at very early epochs.

8 Cosmic background multiconversion by cosmological and galactic magnetic fields

A scenario with the conversion process (EW $\to$ GW) playing a relevant physical role has been pointed out by Zel'dovich [8] in cosmology. He considered coherent conversion of the background radiation (CBR) into GW (the opposite conversion is clearly negligible) by a cosmological magnetic field $B_0 = 10^{-9} \div 10^{-6}$ Gauss. The author [8] studied a refractive medium but took into account only single conversion (where the growth is quadratic or square sinusoidal with the distance and has an upper limit $\simeq p^2/r^2$; see Eq. (40)). In particular, at the redshift $z = 10^3$, for a cosmological magnetic field $B_0 = 1$ Gauss, in the presence of a baryon density $n_a = 10^3 \ cm^{-3}$, a plasma density $n_e = 10^{-1} cm^{-3}$ and for a characteristic EW wave vector $k = \omega/c = 10^4 \ cm^{-1}$, Zel'dovich found

$$\alpha_{\max} \simeq 4p^2/r^2 = 4 \cdot 10^{-12}. \tag{62}$$

He concluded that $\alpha_{\max}$ in Eq. (62) is an absolute upper limit for GW $\leftrightarrow$ EW conversion. However, this result holds only for one oscillation over a coherence

length $L_{\rm coh}$:

$$L_{\rm coh}(p<|r_e|)=\frac{4\pi}{|r|}\frac{\omega}{c}=16\ \frac{\omega}{3\cdot 10^{14}{\rm Hz}}\left(\frac{n_e}{10^{-1}cm^{-3}}\right)^{-1}\ {\rm l.y.} \tag{63}$$

The cosmological age at recombination near the redshift $z=1000$ is $t=t_0(1+z)^{-3/2}\simeq 5\cdot 10^5$ yrs. Therefore the cumulative multiconversion (see Section 7) takes place nealy $3\cdot 10^4$ times and leads to a total conversion factor

$$\alpha\simeq\cdot 10^{-7}. \tag{64}$$

This value is still small but almost at a detectable level. However, it may seem exaggerated (and unrealistic) to consider such a present primordial coherent cosmological magnetic field $B_0=10^{-6}$ Gauss. It is, on the contrary, quite realistic to consider an incoherent magnetic field at smaller (galactic) scales, actually the observed random galactic field, at values of $10^{-6}\div 10^{-5}$ Gauss. At redshift $z=10^3$, $L_{\rm coh}\simeq 10$ l.y., a value which is just comparable with the observed homogeneous scale in the Galaxy and the coherence lengths derived for the refractive index in Eq. (63). Therefore the conversion factor in Eq. (64) is realistic, related to the inhomogeneous random galactic field of the interstellar space at recombination. The primordial galactic contrast over angular scales $\theta\simeq 3^{"}$, needed for adiabatic galaxy formation, is in CDM models of the order

$$\left.\frac{\Delta T}{T}\right|_{z=10^3}\geq 10^{-6}. \tag{65}$$

However, this inhomogeneity has not yet been observed; one possibility is that random multiconversion of the 3 K CBR into GBR leads to a smoother random temperature contrast:

$$\left.\frac{\Delta T}{T}\right|_{z=10^3}=N\frac{p^2}{r^2}\pm\sqrt{N}\frac{p^2}{r^2}\simeq 10^{-7}\pm 10^{-9} \tag{66}$$

where we assume an average present-day galactic magnetic field to be $3\cdot 10^{-6}$ Gauss. Moreover, the conversion of photons into gravitons may deplete the original spectrum leading to a smaller effective CBR at higher frequency. A relevant consequence of photon-graviton multiconversion is therefore a possible presence of a comptonization factor y at a level of few 10^{-7}, much smaller (yet) than the present bounds by COBE ($y\sim 10^{-5}$).

9 Conclusions

As mentioned in the Introduction, the best astrophysical opportunity to test the GW $\leftrightarrow$ EW conversion lies in the huge GW release by supernovae at kilohertz (or tens of kilohertz) frequencies when these waves cross the nearest local magnetic field. Let us assume a nominal SN explosion at 50 kpc whose total GW energy is

10^{51} erg (1% of the neutrino burst). The corresponding flux energy is $\Phi = 4 \cdot 10^3$ erg cm^{-2}. Under this assumption the total GW $\to$ EW conversion may occur either by the terrestrial, Jovian, solar, interstellar, intergalactic magnetic fields, or in SN or its surroundings, either coherently or, more often, incoherently, following the discussion of Eq. (55):

$$
\begin{aligned}
\alpha_{\oplus +} &\simeq 2 \cdot 10^{-32} \Big(\frac{B}{0.5\mathrm{Gauss}}\Big)^2 \Big(\frac{L}{10^4\mathrm{km}}\Big)^2, \\
\alpha_J &\simeq 1.3 \cdot 10^{-28} \Big(\frac{B}{4\mathrm{Gauss}}\Big)^2 \Big(\frac{L}{10^5\mathrm{km}}\Big)^2, \\
\alpha_\odot &\simeq 1.3 \cdot 10^{-26} \Big(\frac{B}{4\mathrm{Gauss}}\Big)^2 \Big(\frac{L}{10^6\mathrm{km}}\Big)^2, \\
\alpha_{\mathrm{i.g.inch}} &\simeq 8 \cdot 10^{-16} \Big(\frac{B}{10^{-6}\mathrm{Gauss}}\Big)^2 \frac{L}{100\mathrm{kpc}} \, \frac{L_c}{100\mathrm{pc}}, \\
\alpha_{\mathrm{i.g.coh}} &\simeq 8 \cdot 10^{-13} \Big(\frac{B}{10^{-5}\mathrm{Gauss}}\Big)^2 \Big(\frac{L}{100\mathrm{kpc}}\Big)^2, \\
\alpha_{NS} &\simeq 8 \cdot 10^{-12} \Big(\frac{B}{10^{12}\mathrm{Gauss}}\Big)^2 \Big(\frac{L}{10^2\mathrm{km}}\Big)^2.
\end{aligned} \tag{67}
$$

The coherence size of galactic magnetic fields has been discussed above (see Eqs. (54),(55)). However, the last and largest conversion for the SN occurs in an enriched plasma arund the SN, so the EWs will be easily screened at those kilohertz radio frequencies. The possible energy enhancement by inverse Compton scattering of GeV electrons by SN and the possible emission of consequent shorter waves could be able to overcome the refraction and reflection in the ionized domains, but these signals will not be discussed here. Therefore we shall neglect the role of the kilohertz GWs converted near a SN. For the same reason we may neglect the GW $\to$ EW conversion near our Sun. We may consider as first conversion to be near (outside) the Earth, i.e., the earliest to be observable, in this case the total energy flux will be

$$
\Phi_{\mathrm{EW}} = \int \frac{d\Phi}{d\omega} d\omega = 8 \cdot 10^{-29} \Big(\frac{B}{B_\oplus}\Big)^2 \Big(\frac{L}{10^4\mathrm{km}}\Big)^2 \;\frac{\mathrm{erg}}{cm^2}. \tag{68}
$$

The total spread of the GW frequency, being probably characterized by a flat step spectrum, falls within $10^4 \div 10^5$ Hz band and the consequent flux is:

$$
\frac{d\Phi_{\mathrm{EW}}}{d\omega}\Big|_{SN1987A} = 8 \cdot 10^{-33} \frac{\mathrm{erg}}{cm^2\,\mathrm{Hz}} = 8 \cdot 10^{-4} \;\mu\mathrm{Jansky}, \tag{69}
$$

too low to be observable. For a 10 kpc source the differential flux would be much larger:

$$
\frac{d\Phi_{\oplus\ \mathrm{EW}}}{d\omega}\Big|_{10\mathrm{kpc}} = 2 \cdot 10^{-2} \;\mu\mathrm{Jansky}. \tag{70}
$$

For the same source a better observational place is near Jupiter's orbit where the total flux would be nearly 6400 times greater, leading to a radio burst:

$$\frac{d\Phi_{\mathrm{J\ EW}}}{d\omega} = 1.38 \cdot 10^2\ \mu\mathrm{Jansky} = 0.14\ \mathrm{mJansky}, \tag{71}$$

within the present radio sensitivity ranges.

Finally we shall reject the overoptimistic case of a total coherent intergalactic field and restrict ourselves to the more modest (but more realistic) case of an incoherent field at 100 pc coherence length. In that case, nevertheless, for a source like SN 1987A, the total energy flux will be still impressive:

$$\frac{d\Phi_{\mathrm{EW}}}{d\omega} = 3.2 \cdot 10^{-16}\ \frac{\mathrm{erg}}{cm^2\,\mathrm{Hz}} = 3.2 \cdot 10^7\ \mathrm{Jansky}. \tag{72}$$

Unfortunately, the flux will not be a longer "prompt" one (as is near Jupiter) but is very much delayed in time because those low energy "photons" behave like massive particles with a relativistic Lorentz factor

$$\Gamma_\gamma = \frac{E_\gamma}{E_{\mathrm{pl}}} \simeq \frac{\omega}{\omega_p} \simeq 10\ \frac{\omega}{10^5\mathrm{Hz}}\ \frac{n_e}{1\ cm^{-3}}. \tag{73}$$

Consequently, the "fastest" energetic EWs (possibly at 30–100 kHz for neutron-star or black-hole sizes) will reach the Earth with a large time delay τ_d with respect to the prompt SN events:

$$\tau_d = \frac{1}{2\Gamma_\gamma^2}\frac{L}{c} \simeq 750\ \frac{L}{1.5 \cdot 10^5\mathrm{yrs}}\left(\frac{\omega}{10^5\mathrm{Hz}}\right)^{-2}\ \mathrm{yrs}. \tag{74}$$

This delay would introduce a huge time dilution of the signal, as well as a severe flux reduction

$$\frac{d\Phi_{\mathrm{EW}}}{d\omega\ dt} \simeq 10^{-3}\ \mathrm{Jansky} \tag{75}$$

in the above estimates. Moreover, a large "refractive" index will give life to some random walk of the radio signals and will smear out their directionality more and more, leading to a further spread and dilution of the signal/noise ratio. Neverthless, the tens-of-kilohertz radio wave band is a very exciting "astrophysical" band to be considered for discovering the secret of gravitational waves. Unfortunately, noise is large and little is known to the author at these bands. These frequencies are actually already used to discover other kinds of secrets. The same satellites which probably look into the deep blue sea at those frequencies might have already recorded, without being aware of it, both the prompt and the delayed signals due to SN 1987A GW → EW conversion (if their sensitivity is above the noise). Finally, it sounds ironically that these military satellites could hide in their recorded data at the lowest radio energies just the exciting secrets of the most powerful explosions in our Galaxy.

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